APPLICATION OF HIGH-SPEED COMPUTING

IN AERONAUTICAL RESEARCH

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Introduction

The application of high-speed computing in aeronautical research is in the earliest stages and the unclassified aeronautical literature contains relatively few papers in which high-speed computing has actually been used. One classical paper is that by W. F. Cope and D. R. Hartree on "The Laminar Boundary Layer in Compressible Flow" (reference 1). This paper contains solutions of an important problem in aeronautical theory obtained on the ENIAC, the first automatic high-speed digital computer installed at Aberdeen Proving Ground. The ENIAC has been applied to many ballistic and guided missile problems, but for the most part the results have appeared in classified publications of limited distribution.

This paper describes chiefly the experience of the National Advisory Committee for Aeronautics, a user rather than a developer of computing machines, and on the basis of that experience attempts to assess the future applications of the more modern equipment in aeronautical research. The NACA has had about three-and-a-half years experience with the Bell Relay

Computer on some 275 aeronautical research problems, and somewhat less experience with the REAC differential analyzer, and with IBM punched card computing equipment. Our research scientists are looking forward to the availability of thoroughly reliable and flexible large scale high-speed digital computers for application to their problems.

The Role of Computation in Aeronautical Research

The task of aeronautical research is a dual one, comprising both the experimental exploration of the physical phenomena encountered in the design and operation of aircraft of all types, and the analytical formulation of "theories" which are in agreement with the observed experimental data and which are rational in character so that reasonable extrapolation can be made to other experimental conditions. Such a rational theory, adequately verified by experiment, is the most powerful tool for the design of aircraft; for the designer may use mathematical analysis rather than costly experimental construction to try the effects of various design variables and in fact to develop an optimum design, which then can be constructed with assurance as to the probable performance. These general considerations apply, whether the problem relate to aerodynamics, stability and control, power plant, or structure.

In this task of aeronautical research, mathematics is the colleague of physics, chemistry, and engineering and with them finds in aeronautics a fertile source of new problems. The indebtedness of aeronautics to mathematics is very great. We are here concerned with that part of mathematics

having to do with numerical computation. This art contributes to both aspects of the aeronautical research task and to their amalgamation into design procedures.

Aeronautical research involves a large amount of data reduction. Whether the experimental investigation is conducted in a wind tunnel, on a piloted aircraft in free flight, on a rocket-propelled missile by observations from the ground or by telemetering, on a jet engine in the laboratory or in the air, or on a structural test specimen, the immediate product is a large mass of raw data in the form of printed tapes, oscillograph records, photographs of instruments or other photographic records, and recorded numerical data. These records must be converted to a form adapted to the computational equipment, tare corrections made, scale and calibration factors applied, various other corrections made, and the resulting values combined in various ways to give the desired coefficients or standardized values. From the mathematical point of view the computational processes are simple, but the number of computing steps for each column of raw data is relatively small. The number of data columns may be quite large, since the large investment of time and money in an experimental airplane or power plant, a test missile, or a model for a large supersonic wind tunnel, necessitates extensive instrumentation to obtain as much information as possible in a single period of operation. In my opinion the requirements for rapid data reduction can be met only by a coordinated system of instrumentation and

computing equipment, not by a consideration of computing equipment alone. The over-all accuracy of the final results is of first concern, since as previously noted the objective of aeronautical research is to determine accurately the differences between the observed experimental data and values predicted from the available theories. The accuracy is generally limited by the sensing and recording devices; it is rare that a computational accuracy of more than four figures is warranted.

The theoretical aspects of aeronautical research are greatly dependent on mathematical analysis and computation. It is true that there are still some areas such as turbulent fluid motion where the key physical principles are still not sufficiently clear that the problems can be reduced to definite mathematical problems. But for the most part there are wide areas in which the fundamental equations are known but cannot be solved. In many of these instances numerical methods are applicable, but the time required by manual methods of computation is so great that the method is impractical for general application. The aeronautical scientist would like to carry out computations with varying degrees of simplification and to investigate by computation the relative importance of the variables involved. I believe that theoretical research is most fruitful when carried out in close relation to corresponding experimental investigations. When discrepancies appear, both experimenter and theoretical worker seek the explanation. The theoretical worker investigates the effect of including factors originally thought

to be secondary, and the planning of the next step depends on the results of the last one. For this reason I believe that this type of computation and the experimental work should be housed in the same laboratory so that the contact may be as close and frequent as possible.

The development of high-speed computing machines provides a powerful new tool to the theoretical worker. Old tasks can be accomplished faster, more reliably, and more completely and new tasks hitherto impractical can be undertaken.

Once a theory has been definitely verified and considered appropriate, it becomes available for use in design. Unless, however, the computations are of extremely simple character, the theory remains of limited practical use until numerous calculations have been made and the results incorporated in design charts or tables. The NACA Tables of Wing-Aileron Coefficients of Oscillating Air Forces for Two-Dimensional Supersonic Flow, that were prepared by use of the Bell Computer at Langley is a typical example (reference 2). The necessity for this type of work can be expected to expand as the theories become more complex and take into consideration additional secondary factors. The value which is ultimately derived from a theory which has been experimentally proven to be correct may well depend ultimately on the degree to which its application can be simplified by the use of design charts. Work in this category is exceptionally well adapted to high-speed computing, and does not normally need to be conducted at an

experimental laboratory, although coordination with both scientist and designer are needed to make the result most useful.

Typical Aeronautical Research Problems Requiring Extensive Computation

As a prelude to a more general discussion, it may be instructive to consider a few typical aeronautical research problems requiring extensive computation.

Effect of Airplane Stability on Gust Loads. - When an airplane encounters a gust, the resulting load on the airplane depends on many factors in addition to the gust velocity distribution in the atmosphere. One important set of factors is the stability characteristics of the airplane. The Bell Computer has been used to study the effects of various airplane stability parameters on the gust loads. Step-wise numerical solutions, derived in matrix notation, have been obtained from the equations of airplane motions in three degrees of freedom--vertical, pitch, and elevator displacement--with forward-speed variations being neglected.

Transonic Dynamics Problem. - Prior to the successful flights of the X-1 through the speed of sound, it was believed that a pilot might have difficulty flying in the transonic range since tests of models by the wing flow method indicated rather large trim changes and poor control effectiveness. A proposal was made that the effects might be ameliorated by accelerating through this speed range, and the problem was to investigate this proposal. The three longitudinal equations of motion were involved with the additional

complication that the problem was of non-linear character since the forces
and moments are functions of the Mach number (ratio of the speed of the air-
plane to the speed of sound). The problem was amenable to solution by the
step-by-step computation of time histories. However, manual computation
required about 100 to 400 hours per case. This problem was solved on the
UCLA differential analyzer in about one percent of the time estimated for
manual computations.

Automatic Pilot Time Lag Problem. - Theoretical investigations have
shown that automatic stabilization can improve the dynamic lateral stability
of airplanes designed for high-speed high-altitude flight, but it is essential
that the automatic pilots respond to the error signal without lag. In order to
determine the effects of this time-lag on dynamic lateral stability, calcula-
tions of motions were made on the Bell Computer for a hypothetical high-
speed airplane equipped with an automatic pilot sensitive to the yawing ac-
celeration of the airplane. This automatic pilot was geared to the rudder so
that the yawing moment at any instant of time was proportional to the yawing
acceleration at some previous instant of time, i. e. displaced by a constant
time lag. The problem involved several solutions, by the step-by-step
Kutta three-eighths rule (reference 3) of the equations of motion of an air-
plane. They are three second-order linear differential equations with con-
stant coefficients, and are coupled with the equation of the automatic pilot
which takes into account constant time lag.

Supersonic Drag and Lift Distribution and Downwash. - A method has
been developed by Lagerstrom (reference 4), applied by Doris Cohen (refer-
ences 5 and 6), and by Frick (reference 7) for computing the characteristics
of wings and control surfaces and for the computation of downwash at super-
sonic speeds, which superposes on a basic conical flow solution other solu-
tions for cancellation of lift beyond the wing contour. In most cases this
superposition is an integration process and usually, because of the form of
the basic conical flow solution, the integration cannot be made analytically.
Such computations are adaptable to a digital computer.

Landing Gear Behavior. - Theoretical calculations were made on the
Bell Computer of the vertical displacement, velocity, and acceleration of
the landing gear during landing impact. The calculations were obtained by
step-wise solutions by the modified Heun method (reference 8) of two sec-
ond order non-linear differential equations.

Coupled Mode Flutter Analysis. - The problem of avoiding flutter,
a self-induced catastrophic vibration of the aircraft structure, requires ex-
tensive numerical computation which can be facilitated by high-speed com-
puting machines. In the Rayleigh type flutter analysis, the assumption is
made that the flutter mode may be approximated by the use of a finite
number of terms of a series of certain selected modal functions. The ac-
curacy of the result depends, in general, on the choice of the modal shapes
that make up the series and on the number of terms used. These shapes

are taken either as the coupled or the uncoupled modes of oscillation of the
system in a vacuum. In an effort to ascertain the merits of using coupled
modes in flutter analysis, a theoretical investigation of the flutter of a uni-
form wing carrying a concentrated weight was made with the aid of the Bell
Computer (reference 9). Since coupled modes were being utilized, two
equations arose for each of the twelve spanwise stations considered on the
wing--one expressing the deflection in bending, the second that in torsion
at the station. Solutions of the resulting sets of twenty-four simultaneous
frequency equations were obtained by iteration.

Aerodynamic Influence Coefficients. - High speeds in aircraft and
missiles have brought the necessity of simultaneous consideration of inter-
actions between aerodynamic loads and structural deflections, called aero-
elastic effects. Recent works on aeroelastic phenomena have indicated that
aerodynamic influence coefficients should yield superior results to those
furnished by the modified strip theory. Aerodynamic influence coefficients
for subsonic speeds, therefore, are being calculated on the Bell Computer
by solving Weissinger's integral equation for the spanwise lift distribution
in the case of several plan forms and for angle-of-attack distributions given
by Green's function. The integral equation is approximated by a matrix
equation, the singularities being taken care of in a separate closed-form
solution. The computation consists of calculating the kernel for each of
the plan forms, multiplying and adding the kernel matrix by and to certain
constant matrices, and of inverting the final matrix.

Heat Transfer. - The problem of aerodynamic heating will require a thorough study of the boundary-layer characteristics under varying conditions of Mach number and rate of heat transfer. Cope and Hartree in the paper previously cited (reference 1) have indicated the computing technique that might be applied to this case. In this field, the use of rapid computing machines is an absolute necessity.

This brief list gives some idea of the scope and variety of the applications of high-speed computing to aeronautical research. A more adequate review would include many additional problems related to the general theories of compressible fluid flow at subsonic, transonic, and supersonic speeds; to the stability and control of aircraft and missiles; to vibration and flutter of aircraft propellers, turbines, and compressors; to the boundary layer flow of fluids; to heat transfer; to aeroelastic phenomena; and to structures involving both elastic and plastic deformation.

General Mathematical Aspects of Aeronautical Research Problems. - From the mathematical point of view, the problems arising run the gamut from the reduction of large quantities of numerical data by simple arithmetical processes, to interpolation, differentiation, integration, numerical solution of linear and non-linear differential equations, transcendental equations, linear simultaneous equations in many (24 or more) unknowns, determining roots of polynomials of high order, etc. For the linear problems the exact mathematical solution is often obtainable in analytic form and the extraction

of numerical results is straightforward, though laborious. For the non-linear problems the exact solution is often unobtainable and recourse must be had to numerical approximation.

The linear problems are often idealizations of the actual non-linear ones. Thus probably the simplest problem, from the mathematical point of view, is that of the dynamic motions of an aircraft. If the aircraft is regarded as a rigid body, its motion is described by the six classical equations for the six degrees of freedom. Non-linearities enter these equations because of the presence of products of the velocity components when the coordinate axes are fixed in the moving body and because the aerodynamic forces and moments are non-linear functions of the velocity components.

With the advent of flight at transonic and supersonic speeds linear theory is being found inadequate. The aerodynamic forces and moments exhibit such large variations with speed in the transonic range that they cannot be approximated linearly even for small perturbations. The elastic deformation of the wings interacts with the aerodynamic loads. The complete treatment of the aeroelastic problem leads to partial differential equations. Plausible simplifying assumptions may permit the problem to be held in some instances within the domain of ordinary differential equations.

Another pressing problem is that of the general theory of compressible fluid flow. The completely general problem involving viscosity and,

in the supersonic case, curved shock waves, is so complex that no adequate method of attack has yet been devised. Under certain simplifying assumptions, portions of the problem can be isolated and solved. If viscosity and curvature of the shock waves are both neglected-- steady, potential flow-- the problem reduces to the solution of a single non-linear partial differential equation. For supersonic flow about thin wings, the problem can be linearized, and explicit solutions can be obtained, usually in the form of multiple definite integrals for certain broad classes of plan forms.

If the flow problem is simplified by restricting attention to two-dimensional fields, the non-linear inviscid case can be treated for super-sonic speeds by the well-known method of characteristics for hyperbolic equations. This is a step-by-step non-iterative procedure which is capable of mechanization. In the subsonic case the equation is elliptic and leads to the necessity of solving a large system of simultaneous algebraic equations. If the well-known relaxation methods can be successfully mechanized without building machines of fantastic capacity and cost, then this case should also be solvable.

Unsteady phenomena of a random nature, such as atmospheric gusts, turbulence, and buffeting lead to the field of statistics. Fourier analysis of random processes should be a valuable tool when the problem of data transcription from oscillograms to digital input is satisfactorily solved.

It is believed that the computational problems which arise in aero-

nautical research do not fall into any single classification. For general use, the most flexible computing machine available would best serve the need, but applications will be found where special purpose machines are economically justified.

NACA Experience with High-Speed Computing

In the fall of 1944 the NACA placed an order with the Western Electric Company for a relay-type computing machine representing a further development of a machine previously developed by Bell Telephone Laboratories for the Anti-Aircraft Artillery Board at Camp Davis. Construction was completed late in 1946. The machine uses teletype apparatus and electrical relay control circuits of the type found in automatic telephone systems. It has a storage capacity of 30 seven-decimal digit numbers in storage counters, a transfer time of 0.07 sec., a speed of 0.3 sec. for addition, and 1 sec. for multiplication. Although extremely slow, as compared to the newer electronic machines, the Bell Computer operating 24 hours a day-- a good part of which is unattended--produces work that is equivalent to the output of about 100 manual computers. Elaborate checking circuits have proven to be almost infallible in insuring accuracy. During three years of operation, the machines have been detected in producing only two arithmetically incorrect answers; these were probably due to two troubles appearing simultaneously.

A Reeves Electronic Analogue Computer has been in use at the

Ames Laboratory for about a year. It is primarily a machine for solving systems of ordinary differential equations, linear and non-linear. The coefficients can be constant or variable, if expressible in analytical or graphical form as functions of a single variable. It has been applied to such problems as time histories of dynamic motions of an airplane, both stable and unstable; velocity profiles in a laminar boundary layer; and supersonic flow of a gas around cones using the exact potential theory.

A special purpose analogue computer, the Philbrick engine controls simulator, is in use at the Lewis Flight Propulsion Laboratory for studying problems relating to the automatic control of jet engines.

The NACA has also had access to the use of mechanical differential analyzers and to various IBM punched-card computing equipment including the Card-Programmed Electronic Calculator. When suitable advanced designs of self-sequencing electronic calculators are available, a machine of this type will be acquired, but to date the NACA has had no direct experience with this type of equipment.

Some statistical data, whose significance is perhaps debatable, may be of interest. In a survey made sometime ago, about five percent of the total NACA manpower,or about 350 persons, were classified as computers, including those engineering aides engaged in computing. Their effort was roughly equally divided between analytical work, data reduction, and data handling, such as reading film, tapes, etc. and

plotting. The Langley Laboratory with 66 computers engaged in analytical work had 76 different problems either in progress or to be done within the near future. Of these problems, 18 percent (or 14) were obviously suitable for a large-capacity high-speed computing machine and were scheduled for the Bell Computer. Another 29 percent (or 22) were conditionally suitable in the sense that if solutions were desired for a sufficient number of cases, the use of a large-capacity high-speed machine would be warranted. The remaining 53 percent (or 40) of the problems were not suitable for a large-capacity high-speed machine because the problems were simple or non-repetitive in character. Only a few problems handled by the data reduction group were found suitable for a high-speed machine. I believe that this is a situation peculiar to a research operation; a laboratory engaged in a standardized testing of development models would probably find more use for high-speed computing in its data reduction.

An analysis was also made of the type of equations involved. Of the 36 problems considered suitable or conditionally suitable for high-speed machine computation, 7 involved first and second order differential equations, mostly linear; 3, third order linear and non-linear differential equations; 1, multiple integrals; 4, ordinary integrals; 8, ten or more simultaneous equations or determinants of higher than the ninth order; 1, a smaller number of simultaneous equations; 2, equations of more than the 3rd degree; 1, transcendental equations; 9, general algebraic equations.

Five to seven significant figures were used to obtain the desired three or four significant figures in the final results.

The prospective user will wish to know which machine is "best" for his particular problem, and in particular, whether a digital computer, or analogue computer is "best". Much more experience will be required to give answers to such questions. We have found in the single field of aeronautical research useful applications of all types, and there remain some computing tasks where manual computation is most appropriate.

The advantages to be realized by the utilization of high-speed automatic computing equipment have been frequently mentioned. They are the same in aeronautical research as elsewhere: Speed, economy, more complete survey possible, and reliability, especially if checking devices are built-in. I do not think any type of computing machine can substitute for experimental equipment such as wind tunnels, experimental airplanes and missiles, or altitude test tanks, but there are some areas in which the reliability of a theoretical computation is greater than that realizable in actual experiment, for example, the computation of the velocity distribution within a laminar boundary layer at supersonic speed.

Outlook for the Future

During the past three or four years a few papers (references 10 - 15) have been published in the Quarterly of Applied Mathematics and in the Journal of the Aeronautical Sciences dealing with the application of punch card machines to the solution of the two-dimensional compressible

flow problem, to the torsion of structures, and to the flutter problem, and with the application of analogue computers to structural problems and to flutter as well as other aeroelastic problems. Several aircraft companies are known to be applying high-speed computing machines to various stability computations and to structural design problems. Based on NACA experience and a general knowledge of work in progress elsewhere, the outlook is for a more intensive use of high-speed automatic computing machines. Most of the using agencies are somewhat conservative and will expect demonstrated reliability and accuracy in the new machines as they now do for example in a new instrument or a new wind tunnel.

The significant factors in the increased demand have already been described. They are the increasing number of non-linear problems, the increasingly complex theories needed to accurately describe the interrelated aerodynamic and structural behavior of aircraft at high speeds, and the necessity for considering the effects of aerodynamic heating in high-speed missiles. All these factors are intimately related to the increasing speeds of aircraft and missiles associated with the development of jet and rocket propulsion and advances in transonic and supersonic aerodynamics. Aeronautical research scientists are eagerly awaiting the development of the modern fast, reliable, and flexible digital computer.

Acknowledgments

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